March 5, 2013

Dear Professor Stanley:

A few months ago I was preparing a talk for our seminar here at Minnesota and noticed that my strict growth paper could have been improved with the introduction of the following operator E, which I hope can be used to get better growth results. Let P be an r-differential poset, and consider the map

$$\underbrace{1 - \frac{UD}{r} + \frac{U^2D^2}{r^2 2!} - \ldots \pm \frac{U^nD^n}{r^n n!}}_{E_n} : \mathbb{Z}P_n \to \mathbb{Z}P_n. \tag{1}$$

This operator has the following properties:

(a) $DE_n = 0$,

(b) $E_n \neq 0$ for $n \geq 1$.

In particular,

$$1 \leq \dim \operatorname{Im} E \leq p_n - p_{n-1}. \tag{2}$$

Property (a) is easy: write $DU^i = riU^{i-1} + U^iD$, so that DE telescopes. Property (b) follows from the fact that $E(t_n) \neq 0$ for $n \geq 1$, where

$$\hat{0} = t_0 \prec t_1 \prec t_2 \prec \ldots$$

is a saturated chain such that $Dt_i = t_{i-1}$ for $i \geq 1$. This uses another such chain

$$\hat{0} = s_0 \prec s_1 \prec s_2 \prec \ldots$$

where $s_2 \neq t_2$, so that $s_i \neq t_i$ for $i \geq 2$; see my strict growth paper arXiv:1202.3006.

Remark. Interestingly, (a) is a consequence of the following remarkable formula which is proven in the same way. Here t is a variable.

$$(UD_n + tI)^{-1} = \sum_{i=0}^{n} (-1)^i \frac{U^iD^i}{t(r+t)\cdots(ri+t)} \tag{3}$$

The pair of chains mentioned above may be used with this formula to show that for any $k \in \mathbb{Z}$, the last Smith entry of $UD_n + kI$ is the product of the set of eigenvalues of $UD_n + kI$, just as predicted by the main conjecture that Vic and I made: $DU + tI$ and $UD + tI$ have Smith normal forms over $\mathbb{Z}[t]$.

Best wishes,

Alexander R. Miller

cc: Professor Zanello